PRE-ALGEBRA

Pre-Algebra MID-YEAR TEST + Study Guide!

Created by:
ALL THINGS ALGEBRA

Pre-Algebra Mid-Year Test Study Guide

Name: _____

Date: _____ Per: _____

Topic A: Exponents, Scientific Notation, and Roots

Negative Exponent Rule: $x^{-a} = \frac{1}{x^a}$

Zero Exponent Rule: $x^0 = 1$

Rewrite the expressions using only positive exponents. Simplify if possible.

1. 3^{-3} $\frac{1}{3^3} = \frac{1}{27}$

2. $2^{-7} \cdot 12^2$ $\frac{12^2}{2^7} = \frac{144}{128} = \frac{72}{64} = \frac{9}{8}$

3. $9^{-1} \cdot 4^0 \cdot 4^{-3}$ $\frac{1}{9 \cdot 4^3} = \frac{1}{9 \cdot 64} = \frac{1}{576}$

Write the following values in scientific notation.

4. 27,500 2.75×10^4

5. 0.0016 1.6×10^{-3}

6. 0.0000000047 4.7×10^{-9}

Write the following values in standard form.

7. 2.92×10^{-1} 0.292

8. 5×10^7 50,000,000

9. 6.3×10^{-5} 0.000063

Evaluate each expression.

10. $\sqrt{81}$ 9

11. $-\sqrt{324}$ -18

12. $\sqrt{\frac{1}{4}}$ $\frac{1}{2}$

13. $\sqrt[3]{8}$ 2

14. $\sqrt[3]{-343}$ -7

15. $\sqrt[3]{1{,}728}$ 12

16. Which values are **perfect squares?**

20 (121) (36) 10 18 (256) (400)

17. Which values are **perfect cubes?**

(64) 9 300 (27) (512) (1) 250

Estimate the following values to the nearest tenth.

18. $\sqrt{105}$

19. $\sqrt{68}$

20. $-\sqrt{236}$

Determine the integers between which each square root lies.

21. $\sqrt{14}$ $3 < \sqrt{14} < 4$

22. $-\sqrt{84}$ $-10 < -\sqrt{84} < -9$

23. $-\sqrt{192}$ $-14 < -\sqrt{192} < -13$

Topic B: The Real Number System

THE REAL NUMBERS: all numbers

IRRATIONAL NUMBERS: non repeating decimals π $\sqrt{2}$ $\sqrt[3]{5}$

RATIONAL NUMBERS: anything that can be a fraction

INTEGERS: $\{\ldots -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, \ldots\}$

WHOLE NUMBERS: $\{0, 1, 2, 3, 4, 5, \ldots\}$

NATURAL NUMBERS: $\{1, 2, 3, 4, 5, 6, \ldots\}$

Name the SMALLEST subset to which each value belongs.

1. $-\frac{13}{18}$ Rational	2. $-\sqrt{289}$ Integers	3. $\sqrt{32}$ Irrational
4. 7^{-2} Rational	5. $\lvert -15 \rvert$ Natural	6. 7.5291 Rational
7. $\sqrt[3]{125}$ Natural	8. $\pi - \pi$ Whole	9. $\frac{-\sqrt{100}}{20}$ Rational

Topic C: Comparing & Ordering Number Forms

Rewrite #1 in order from least to greatest, then #2 in order from greatest to least.

1. $\{\frac{5}{9}, 0.56, 5.5\times10^{-2}, 5^{-1}, 6\%, 5.5\times10^{-1}\}$
   $\frac{5}{9}$ 0.5 | 0.56 | 5.5×10^{-2} 0.055 | 5^{-1} 0.2 | 6% 0.06 | 5.5×10^{-1} 0.55

   5.5×10^{-2}, 6%, 5^{-1}, 5.5×10^{-1}, $\frac{5}{9}$, 0.56

2. $\{\sqrt{144}, 1.2\times10^{2}, \sqrt[3]{729}, 2^{7}, 3^{3}, 9\times10^{1}, \sqrt{100}\}$
   $\sqrt{144}$ 12 | 1.2×10^{2} 120 | $\sqrt[3]{729}$ 9 | 2^{7} 128 | 3^{3} 27 | 9×10^{1} 90 | $\sqrt{100}$ 10

   $\sqrt[3]{729}$, $\sqrt{100}$, $\sqrt{144}$, 3^{3}, 9×10^{1}, 1.2×10^{2}, 2^{7}

Topic D: Order of Operations & Evaluating Expressions

Evaluate each expression.

1. $8^2 \div 4(7-2^5)+7^3$

   $64 \div 4(7-32)+343$

   $64 \div 4(-25)+343$

   $16(-25)+343$

   $-400+343$

   -57

2. $\frac{\lvert -18-6(2) \rvert}{(8-3)^2-1}$

   $\frac{\lvert -18-12 \rvert}{5^2-1} = \frac{\lvert -30 \rvert}{25-1}$

   $\frac{30}{24} = \frac{5}{4}$

3. $\sqrt{225}-4(5-3)^3+\sqrt[3]{27}$

   $15-4(2)^3+3$

   $15-4(8)+3$

   $15-32+3$

   $-17+3$

   -14

4. $1\frac{9}{10} \cdot -\frac{5}{9} + \frac{7}{3}$

$\frac{19}{10} \cdot -\frac{1}{9} + \frac{7}{3}$

$-\frac{19}{18} + \frac{7 \cdot 6}{3 \cdot 6}$

$-\frac{19}{18} + \frac{42}{18} = \frac{-23}{18} = 1\frac{5}{18}$

5. $\left(16 - 10\frac{5}{6}\right) \div -\frac{1}{3}$

$5\frac{1}{6} \div -\frac{1}{3}$

$\frac{31}{6} \cdot -\frac{3}{1} = -\frac{31}{2} = -15\frac{1}{2}$

Evaluate each expression given the replacement values.

6. $\frac{wv^2 - 4v}{2w + 3v}$ if $w = 8$ and $v = -3$

$\frac{8(-3)^2 - 4(-3)}{2(8) + 3(-3)} = \frac{8(9) + (+12)}{16 + (-9)}$

$\frac{72 + 12}{7} = \frac{84}{7} = 12$

7. $|-10 + 7a| - \frac{15}{2}b + 5a^3$ if $a = -2$ and $b = \frac{4}{3}$

$|-10 + 7(-2)| - \frac{15}{2}\left(\frac{4}{3}\right)^2 + 5(-2)^3$

$|-10 + -14| - 10 + 5(-8)$

$|-24| - 10 + (-40)$

$24 - 10 + (-40)$

$14 + (-40) = -26$

Topic E: Properties

Commutative Property (of add. or mult.)
Associative Property (of add. or mult.)
Identity Property (of add. or mult.)
Inverse Property (of add. or mult.)
Property of Zero
Distributive Property

Name the properties that justifies each statement.

1. $15x + (-15x) = 0$

2. $(-7 \cdot -2) \cdot 5 = -7 \cdot (-2 \cdot 5)$

3. $3(8 + 2) = 3 \cdot 8 + 3 \cdot 2$

4. $(m + n) + p = p + (m + n)$

5. $5a^2 \cdot \frac{1}{5a^2} = 1$

6. $(9r + 4s) \cdot 0 = 0$

7. Name the additive identity of $\frac{2}{3}$.

8. Name the multiplicative inverse of $7y$.

Topic F: Simplifying & Factoring Expressions

Simplify the following expressions.

1. $10 - 2k - 5k + 28$

$-7k + 38$

2. $-3a - 7 - 2a + 9a - 16$

$4a - 23$

3. $4y - 2x + 8y - x + 14y$

$-3x + 26y$

4. $3(m-7)$

$3m-21$

5. $-5(2x+7)$

$-10x+(-35)$

6. $-(r-3s)$

$-r+3s$

7. $-4(x-9)+2x-1$

$-4x+36+2x-1$

$-2x+35$

8. $-w+8(3-4w)+19w$

$-w+24-32w+19w$

$-14w+24$

9. $-\frac{1}{2}(2c+18d)-(c-d)$

$-c-9d-c+d$

$-2c-8d$

Factor the following expressions. If it cannot be factored, write "prime."

10. $4x+4$

11. $6r-45$

12. $9x-32y$

13. $32m+24n$

Topic G: Solving Equations

Solve the following equations. Watch out for special solutions!

1. $-3n+17=56$

$-17\ \ -17$

$-3n=39$

$\frac{-3n}{-3}=\frac{39}{-3}$

$n=-13$

2. $-21=-16+\frac{x}{1.8}$

$+16\ \ +16$

$-5=\frac{x}{1.8}$

$X=-9$

3. $\frac{m+8}{-2}=-19$ $\cdot -2$

$m+8=38$

$-8\ \ -8$

$m=30$

4. $\frac{4}{3}k+15=-31$

$-15\ \ -15$

$\frac{3}{4}\cdot\frac{4}{3}k=-46\cdot\frac{3}{4}$

$k=-34.5$

5. $a-16=4a+29$

$-4a\ \ -4a$

$-3a-16=29$

$+16\ \ +16$

$\frac{-3a}{-3}=\frac{45}{-3}$

$a=-15$

6. $8(p-3)=6(2p+5)$

$8p-24=12p+30$

$-12p\ \ -12p$

$-4p-24=30$

$+24\ \ +24$

$\frac{-4p}{-4}=\frac{54}{-4}$

$p=-13.5$

7. $-\frac{1}{4}(20m-8)=4(m-4)$

$-5m+2=4m-16$

$-4m\ \ -4m$

$-9m+2=-16$

$-2\ \ -2$

$\frac{-9m}{-9}=\frac{-18}{-9}$

$m=2$

8. $2(5y-8)=9y-(16-y)$

$10y-16=9y-16+y$

$10y-16=10y-16$

$-10y\ \ -10y$

$-16=-16$

Infinite Solutions

9. $3n+6(n+4)=9(n-2)$

$3n+6n+24=9n-18$

$9n+24=9n-18$

$-9n\ \ -9n$

$24\neq-18$

No Solution

10. Four-fifths of the people that Kyle and Melissa invited to their wedding responded indicating that they are coming. However, of those that responded, 19 did not show up. If there were a total of 121 guests at their wedding, how many did they invite? Let c= those that they invited

$\frac{4}{5}c - 19 = 121$
$+19 \quad +19$
$\frac{5}{4} \cdot \frac{4}{5}c = 140 \cdot \frac{5}{4}$

$c = 175$ invited

11. The sum of two numbers is 58. If the larger number is ten less than three times the smaller number, find both numbers. Let x= the smaller number
Let 3x−10 = the larger number

$x + 3x - 10 = 58$
$4x - 10 = 58$
$+10 \quad +10$
$\frac{4x}{4} = \frac{68}{4}$
$x = 17, 41$

$3(17) - 10$
$51 - 10$
41

12. The seventh and eighth grade students are selling tickets to the talent show. The eighth grade students have sold fifteen more than twice the number of tickets that the seventh grade students have. If they sold 486 tickets altogether, how many did the eighth grade students sell?
Let x= seventh graders
2x+15= eight graders

$x + 2x + 15 = 486$
$3x + 15 = 486$
$-15 \quad -15$
$3x = 371$
$\frac{3x}{3} = \frac{371}{3}$
$x = 127$

$2(127) + 15$

269 tickets

Topic H: Solving & Graphing Inequalities

1. Solve and graph: $4x + 7 \geq -13$
$-7 \quad -7$
$\frac{4x}{4} \geq \frac{-20}{4}$
$x \geq -5$

2. Solve and graph: $\frac{m}{-4} + 3 > 1$
$-3 \quad -3$
$-4 \cdot \frac{m}{-4} > -2 \cdot -4$
$m < 8$

3. Solve and check possible solutions: $14 - 2k < 40$
$-14 \quad -14$
$\frac{-2k}{-2} < \frac{26}{-2}$
$k > -13$

- ☐ -15
- ☐ -14
- ☐ -13
- ☑ -12
- ☑ -11

4. Solve and check possible solutions: $-1 + \frac{5}{2}p \leq 9$
$-1 + \frac{5}{2}p \leq 9$
$+1 \quad +1$
$\frac{2}{5} \cdot \frac{5}{2}p \leq 10 \cdot \frac{2}{5}$
$p \leq 4$

- ☑ 1
- ☑ 2
- ☑ 3
- ☑ 4
- ☐ 5

5. Solve and graph: $11w - 5 \geq 4w - 26$

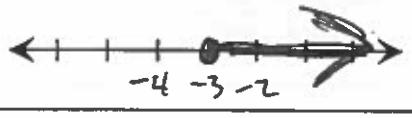
$-4w \quad -4w$

$7w - 5 \geq -26$

$+5 \quad +5$

$\frac{7w}{7} \geq \frac{-21}{7}$

$w \geq -3$



6. Solve and graph: $2(3c+5)+1 > 8c-3$

$6c + 10 + 1 > 8c - 3$

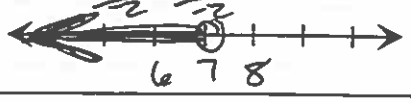
$6c + 11 > 8c - 3$

$-8c \quad -8c$

$-2c + 11 > -3$

$-11 \quad -11$

$\frac{-2c}{-2} > \frac{-14}{-2}$ $\quad c < 7$



Topic I: Translating Equations & Inequalities

Translate each equation or inequality. Do not solve.

1. "Four more than the product of a number and -7 is 39." $-7n + 4 = 39$

2. "The sum of five-sixths of a number and 1 is -14." $\frac{5}{6}n + 1 = -14$

3. "Seventeen less than twice a number is -49." $2n - 17 = -49$

4. "The difference between a number and 10, divided by -4, results in 6." $\frac{n-10}{-4} = 6$

5. "The quotient of a number and -3 is greater than or equal to -7." $\frac{n}{-3} \geq -7$

6. "Nine subtracted from four times a number is less than 19." $4n - 9 < 19$

7. "The product of a number and -5, increased by 8, is at least 53." $-5n + 8 \geq 53$

8. "Fourteen more than twice a number is no more than -6." $2n + 14 \leq -6$

Topic J: Exponent Rules

Product Rule	Quotient Rule	Power Rule
$x^a \cdot x^b = x^{a+b}$	$\frac{x^a}{x^b} = x^{a-b}$	$(x^a)^b = x^{ab}$

Simplify each expression. Your final answer can contain only positive exponents.

1. $x^7 \cdot x^4$ $\quad x^{7+4}$ $\quad x^{11}$

2. $-5a^{-2} \cdot 4a^{11}$ $\quad (-5 \cdot 4)a^{-2+11}$ $\quad -20a^9$

3. $7m^4n^{-2} \cdot 8m^{-6}n^{-1}$ $\quad (7 \cdot 8)m^{4+(-6)}n^{-2+(-1)}$ $\quad 56m^{-2}n^{-3} = \frac{56}{m^2n^3}$

4. $\frac{y^6}{y^8}$ $\quad y^{6-8} = y^{-2} = \frac{1}{y^2}$

5. $\frac{63k^4}{9k^{-1}} = \frac{63}{9}k^{4+(+1)}$ $\quad 7k^5$

6. $\frac{5r^{-3}s^8t^{-1}}{10r^4s^2t^{-1}}\left(\frac{s}{10}\right)$ $\quad r^{-3-4}s^{8-4}t^{-1+(+1)}$ $\quad -\frac{1}{2}r^{-7}s^4t^0 = -\frac{s^4}{2r^7}$

7. $(w^2)^8$ $w^{2 \cdot 8}$ w^{16}

8. $(-6a^4b^9)^2$ $(-6)^2 a^{4 \cdot 2} b^{9 \cdot 2}$ $36a^8b^{18}$

9. $(2c^{-1}d^2)^5$ $2^5 c^{-5} d^{10} = \frac{32d^{10}}{c^5}$

Evaluate each expression. Give all final answers in scientific notation.

10. $5.4 \times 10^3 + 1.9 \times 10^3$

11. $4 \times 10^6 + 2.5 \times 10^7$

12. $9.8 \times 10^{-3} - 3 \times 10^{-4}$

13. $8 \times 10^8 - 2 \times 10^{10}$

14. $(7.5 \times 10^7)(1.2 \times 10^3)$

9.0×10^{10}

Add Exponents

$7.5 \times 1.2 = 150 + 750 = 9.00$

15. $(9 \times 10^2)(8 \times 10^{-7})$

72×10^{-5}

7.2×10^{-4}

Only one # in front of decimal

Left Add

16. $(8.4 \times 10^5) \div (7 \times 10^1)$

1.2×10^4

Subtract Exponents

$7 \overline{)8.4}$ → 7, 14, 14, 0

17. $\frac{1.9 \times 10^{-8}}{5 \times 10^{-2}}$ $-8 + (+2) = -6$

0.38×10^{-6}

3.8×10^{-7}

$5 \overline{)1.9}$ → 0.38, 15, 40

Right Subtract

18. The population of Buffalo, New York is approximately 2.6×10^5 people. If the population of New York City is approximately 8.4×10^6 people, how many more people live in New York City than Buffalo?

84×10^5

$- 2.6$

81.4×10^5

8.14×10^6

19. The mass of a proton is approximately 1.7×10^{-24} grams. Find the mass of 250 protons.

Topic K: Ratios, Rates, & Proportions

Use for questions 1-2:
Jeff took a three-day road trip. The table below shows the number of hours and miles he drove each day.

	Monday	Tuesday	Wednesday
Hours	5	8	4
Miles	264	436	200

1. What is the ratio of miles driven on Tuesday to the total number of miles driven?

$\frac{436}{900} = \frac{218}{450} = \frac{109}{225}$

2. Find the rate at which Jeff was driving on Monday. $\frac{264}{5} = 52.8$ mi/hr

Determine whether Option A or Option B is the better deal. Justify your choice.

3.

- ☐ **Option A:** 6 candy bars for $5.29 0.88
- ☑ **Option B:** 8 candy bars for $7.50 0.94

4.

- ☑ **Option A:** $20 for 15 raffle tickets 0.75
- ☐ **Option B:** $50 for 40 raffle tickets 0.80

Determine if the data in the table represents a proportional relationship. If yes, give the constant rate.

5.

Highway Tolls		
Miles	**Toll**	
15	$0.90	0.06
35	$2.10	0.06
80	$4.80	0.06
95	$5.70	0.06

Yes, $0.06/mi

6.

Class Sizes		
# of Classes	**Students**	
3	96	32
5	160	32
8	232	29
12	312	26

No

7.

Calories in Cupcakes		
Cupcakes	**Calories**	
2	464	232
5	1,160	232
9	2,088	232
15	3,480	232

232 cal/cupcake

Solve each proportion.

8. $\frac{9}{5} = \frac{63}{x}$

$\frac{9x}{9} = \frac{315}{9}$

$x = 35$

9. $\frac{28}{m} = \frac{10}{27}$

$10m = 756$

$m = 75.6$

10. $\frac{a}{13} = \frac{41.8}{71.5}$

$\frac{71.5a}{71.5} = \frac{543.4}{71.5}$

$a = 7.6$

11. Rylan worked 15 hours last week and earned $138. If he works 23 hours this week, how much will he earn?

$\frac{15}{138} = \frac{23}{x}$

$\frac{15x}{15} = \frac{3174}{15}$

$211.60

12. Erin travels a lot for her job. She flew 3,649 miles in January, 2,540 miles in February, and 4,067 miles in March. If the airline awards her 50 points for every 800 miles she travels, how many points did she earn during the three months?

$\frac{50}{800} = \frac{x}{10256}$

$x = 641$

Topic 1: Percents

1. If there are 350 eighth students and 74% returned signed permission slips to attend the end of year trip, how many are going on the trip?

$\frac{x}{350} = \frac{74}{100}$

$x = 259$ students

2. During his gym workout, Kyle burned 327 calories on the treadmill. If this was 37.5% of the total calories burned during his workout, how many total calories did he burn?

$\frac{327}{x} = \frac{37.5}{100}$

$x = 872$

3. During a Black Friday sale, Bryson bought a $679 TV that was on sale for 40% off and a $224 tablet that was on sale for 15% off. How much did he save in total between these two items?

$\frac{d}{679} = \frac{40}{100}$ $\frac{d}{224} = \frac{15}{100}$

$271.60 $33.60

$305.20

4. Keisha is buying a pair of jeans that are regularly $54.99. If they are on the clearance rack for 35% off and sales tax is 7.2%, what is the total amount she will pay?

$\frac{d}{54.99} = \frac{35}{100}$

$d = 19.25$

$54.99 − 19.25 = 35.74$

$\frac{t}{35.74} = \frac{7.2}{100}$

$t = \$2.57$

$+35.74$

$38.31

5. A baseball card that sold for $1,200 in 2004 sold for $850 in 2010. Find the percent of change from 2004 to 2010. Round to the nearest tenth of a percent.

$\frac{1200 - 850}{1200} = \frac{350}{1200} = 29.2\%$ decrease

6. Charlie's dinner bill came to $42.95. If he paid $50, what percent did he tip? Round to the nearest tenth of a percent.

$50.00 − 42.95 = 7.05$

$\frac{7.05}{42.95} = \frac{P}{100}$

16.4%

7. Minimum wage in Vermont was $5.15 in 2000. In 2016, it increased to 70¢ less than twice what it was in 2000. Find the percent of change from 2000 to 2016. Round to the nearest tenth of a percent.

$2x - 0.70$

$2(5.15) - 0.70$

$9.60

$9.60 − 5.15$

$\frac{4.45}{5.15} =$ 86.4% increase

8. Troy put $2,800 in a savings account with a simple interest rate of 1.9%. If he does not touch the money for 20 years, how much will interest will it earn?

9. Scott took out a 72-month loan for $42,000 to put a new pool in his background. If the simple interest rate is 6.75%, how much will he pay in total?

10. A $7,500 investment in 2009 grew to $9,037.50 in 2014. Assuming no deposits were made during this time, find the simple interest rate.